--- Lecture 1 ---

Electric Charge, Coulomb's Law, Electric Field

Electric Charge is an observed (*i.e.* PHENOMENOLOGICAL) fundamental property of matter. Furthermore, two *types* or *flavours* of charge exist: POSITIVE (+) and NEGATIVE (-).]
[In modern parlance, one could call these "CHARGE" and "ANTI-CHARGE", respectively.]
There is no compelling theoretical explanation for the *existence* of electric charge. It is an attribute of the *fundmental constituents* of everyday matter, *i.e.*, those particles [proton, neutron, and electron] which comprise chemical atoms.

Electric charge is, underlyingly, responsible for nearly all of the structure and dynamics that we observe in the world around us. That is: the chemical bonding into compounds of which matter, as we know it, is made is essentially of electric origin, [with important quantum mechanical and subtle magnetic effects responsible for some particulars]. In addition, the *cohesion* of matter into liquid and solid states occurs through residual electro-magnetic (electric) forces. The [APPROXIMATE] *rigidity* of solids which gives rise to phenomenological NORMAL FORCES and FORCES OF CONTACT are also of electromagnetic origin.

With only slight exaggeration -- residuals of the strong nuclear force bind the protons and neutrons into atomic nuclei characteristic of chemical isotopic elements -- we can tease our philosopher friends by averring that electric forces generate *order* and *form* from the *amorphous chaos* of elementary particles.

Being pragmatic physicists, we are content to model the physical effects of electric charge.

Investigations into electricity and magnetism started "*way back*" with the Greek philosopher-scientists who realised that by rubbing things together (silk or fur on glass or amber), they could do *party tricks*.

The history makes for fascinating study and as is so often the case with human endeavour, it brims with drama, serendipity, nobility, concupiscence, inspiration and perspiration. The development of the modern classical theory of electromagnetism, the subject of this course, is a tale full of twists and turns. In this class, we shall eschew the slavishly historical route and present the finished models. We cannot completely avoid the history though, due to the existence of accidents of convention. The best example of which is the definition of electric current promoted by Ben Franklin and accepted by the community of 18th Century scientists as the flux of **positive charge**. Much later, in the final decade of the 19th Century, it was realised that, in most cases, the *microscopic origin* of electric current is the *coherent drift* of negatively charged electrons in the diametrically opposite direction.

Ordinary matter is typically electrically neutral. The generation of **static electric charges** [charges which persist in time] in bulk matter is usually accomplished by the transfer of electrons to or from neutral matter.

- transfer of electrons **to** results in **negative net charge**
- transfer of electrons **from** results in **positive net charge**

Three terms which we must define (qualitatively) are *conductor, insulator*, and *ground*.

- **Conductor:** A material in which charge is easily transported.
- **Insulator:** A material in which charge is NOT easily transported.
- **Ground:** A source of charge which is [essentially] infinite.

The two most important features of electric charge are:

1. **Electric charge is CONSERVED**
Electric charge is neither created nor destroyed. This does not prevent transfer of charge from one location to another, nor the production of equal and opposite charges from neutral matter by polarisation.
2. **Electric charge is QUANTISED**
Macroscopic, or "observable," electric charge is quantised in units of the *fundamental charge* (that which is borne by an electron or proton).
   - **Protons** have charge +*e*.
   - **Electrons** have charge -*e*.
   - **Neutrons** have zero electric charge.
   - **Quarks** and **anti-quarks** have charges $\pm\frac{1}{3}(-e)$, for the "down" family and $\pm\frac{2}{3}e$ for the "up" family. A property particular to the theory of quarks, "*confinement*," ensures that single "bare quarks" may never be isolated, and as a consequence only quark--anti-quark pairs and quark--quark--quark or anti-quark--anti-quark--anti-quark triples can appear as long-lived low-energy propagating-particle states. Thus *all* of the elementary particles which are *observed* [in low-energy "initial" and "final" states] in particle physics experiments bear charges which are in the set { -2*e*, -*e*, 0, *e*, 2*e* }.

More generally, the exact amount of electric charge borne by **any** physical object must be a multiple of the fundamental or "elementary" charge.

$$q = N\,e \text{ for some } N \in \mathbf{Z}.$$

For large values of *N*, the quantum nature of the charge is hard to discern.

The SI unit of charge is the **Coulomb**, "C."

$$e = 1.609 \times 10^{-19}\ \mathrm{C} \approx 1.6 \times 10^{-19}\ \mathrm{C}.$$

There are about $(1.6 \times 10^{-19}\ \mathrm{C})^{-1} \approx 6.25 \times 10^{18}$ *elementary charges* in 1 Coulomb. The immense size of this number provides an illustration of how it is that the granularity of electric charge is not readily apparent on macroscopic, and indeed microscopic, scales, and justifies the *classical* model of charge as a continuous fluid.

ASIDE: The same ideas appear in discussion of the essential granularity of atomic/molecular matter.

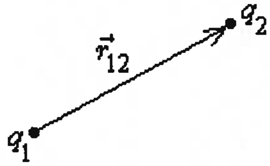
Q: Okay, okay, but HOW do two charges *interact*?
A: [*Colloquial*]
Like charges repel and unlike charges attract.

A: [*Classical*]
There exists a force of interaction between two point charges which acts along the line joining them. This force is called the *Coulomb Force* and its mathematical formulation is termed *Coulomb's Law*, and expanded upon below.

A: [*Modern Viewpoint*]
In QUANTUM ELECTRODYNAMICS (QED), the electromagnetic interaction between electric charges

is mediated by the exchange of VIRTUAL PHOTONS.

Coulomb's Law for two point(-like) charges



The force produced by "the particle bearing charge q_1" -- [q_1 for short] -- acting on "the particle bearing charge q_2" -- [abbreviated to q_2] -- is

$$\vec{F}_{12} = k\frac{q_1 q_2}{|\vec{r}_{12}|^2}\hat{r}_{12}.$$

In this expression the **Coulomb Constant**, $k = 8.99 \times 10^9$ N·m^2/C^2 while $\hat{r}_{12}$ is the unit vector pointing from q_1 to q_2 and furthermore, $|\vec{r}_{12}|^2$ denotes the squared-magnitude of the distance between the charges.

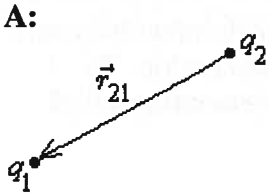
The Coulombic Force is an "inverse-square law."

ASIDE: Coulomb was positively ecstatic when he obtained the inverse-squared law (*circa* 1785), because it precisely mimicked the form of Newton's Law of Universal Gravitation (*circa* 1666). It followed immediately that the many *wonderful* properties [*i.e.*, conservative force, integrability] which had been demonstrated to hold for newtonian gravitation automatically carried over to the Coulombic electric force.

The colloquial description of the interaction of electric charges is consistent with the Coulomb formula.

Like Charges
IF $q_1 \cdot q_2 > 0$, THEN $\vec{F}_{12}$ acts in the direction of $\vec{r}_{12}$ so these charges **repel**.
Unlike Charges
IF $q_1 \cdot q_2 < 0$, THEN $\vec{F}_{12}$ acts in the direction opposite to $\vec{r}_{12}$ so these charges **attract**.

Having determined the force produced by charge 1 acting on charge 2, it is eminently reasonable to ask the following question.
Q: What is $\vec{F}_{21}$?
A:



The force produced by q_2, acting on q_1 is $\vec{F}_{21} = k\frac{q_1 q_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$.
The reversed vector is of equal magnitude, and opposite direction, $\vec{r}_{21} = -\vec{r}_{12}$, and thus

$$\hat{r}_{21} = -\hat{r}_{12}\ ,\ \text{and}\ |\vec{r}_{21}|^2 = |\vec{r}_{12}|^2.$$

Hence, $\vec{F}_{21} = -k\frac{q_1 q_2}{|\vec{r}_{12}|^2}\hat{r}_{12} = -\vec{F}_{12}$, in perfect accord with **Newton's Third Law**.

Next, we might ponder the question,
Q: What happens if there are more than two charged particles present?
A: Invoke the PRINCIPLE OF LINEAR SUPERPOSITION.
The net force exerted by a set of charges -- designated by the collective "q-primed" -- consisting

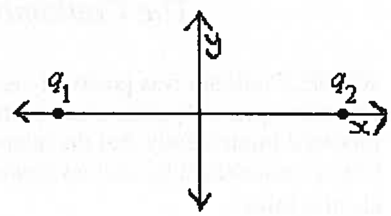
of point(-like) charges q_i, $i = 1,2,\ldots N$, upon a specific charge q_0 is the vector sum of the forces exerted by the q_i's taken individually. That is

$$\vec{F}_{q'q_0} = \vec{F}_{q_1q_0} + \vec{F}_{q_2q_0} + \vec{F}_{q_3q_0} + \ldots = \sum_{i=1}^{N} \vec{F}_{q_iq_0} .$$

Q: Are there any helpful tricks - to spare us tedious computations?
A: Yes. Exploit symmetries wherever possible.

Example [*Coulombic force in very symmetrical situations*]

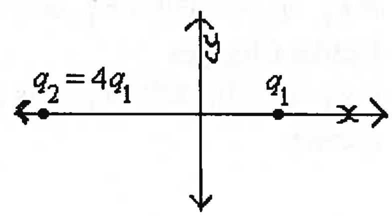
Say that $q_1 = 1\ \mu C$ is at $\vec{x}_1 = (-2, 0)$ and $q_2 = 1\ \mu C$ is at $\vec{x}_2 = (2, 0)$.



Q: What is the **Coulombic Force** exerted on a charge q_0, located at the origin?
A: Zero. In fact, the net x-component is zero all along the y-axis.

Example [*Coulombic force in less symmetrical situations*]

Say that $q_1 = 1\ \mu C$ is at $\vec{x}_1 = (2, 0)$ and $q_2 = 4\ \mu C$ is at $\vec{x}_2 = (-4, 0)$.



Q: What is the **Coulombic Force** exerted on a charge q_0, located at the origin?
A: Zero again. However, everywhere except at the origin, the situation is not so simple.

Coulomb's Law is plagued by the same "*Action at a Distance*" difficulties which infect Newton's Law of Universal Gravitation. There are many philosophical problems with this notion. To ameliorate these difficulties, we introduce the concept of an **Electric Field permeating all of space**.
Formally, we define the Electric Field, $\vec{E}$ by means of a limiting process.

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}_{q'q_0}}{q_0}$$

- Think of q_0 as a "test charge" located at the *point* in space where we want to measure [determine] the field $\vec{E}$.
- The limit $q_0 \to 0$ is taken so that the other charges are not themselves affected by Coulombic forces produced by q_0.

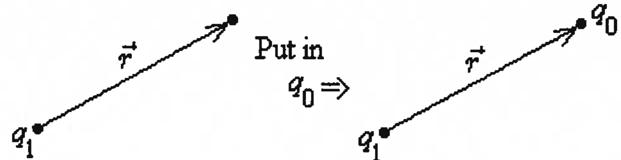
- $\vec{E}$ is a vector field with units Newtons per Coulomb (N/C) which is consistent with the interpretation of the electric field as "*electric force per unit charge.*"

  NOTATION ALERT: A FIELD POINT is a location/position in space at which we are concerned with the electromagnetic goings-on.

The following example is trivial, but we must start somewhere.

Example [*Electric field of a point charge*]

The electric field produced by a single point charge q_1, at a field point P described by the relative position vector $\vec{r}$, is straightforwardly determined by the defining procedure.



$$\vec{F}_{q_1 q_0} = \frac{k q_1 q_0}{r^2_{10}} \hat{r}_{10}$$

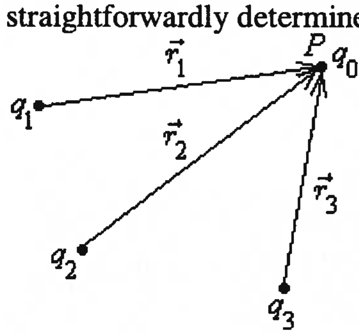
$$\vec{E}_{\vec{r}} = \lim_{q_0 \to 0} \frac{\vec{F}_{q_1 q_0}}{q_0} = \lim_{q_0 \to 0} \frac{1}{q_0} \frac{k q_0 q_1}{r^2_{10}} \hat{r}_{10}$$

$$\vec{E}_{\vec{r}} = \frac{k\, q_1}{r^2_{10}} \hat{r}_{10}$$

Consider the following less trivial example.

Example [*Electric field of a collection of point charges*]

The electric field, produced by a collection of point(-like) charges, at a field point P is also straightforwardly determined.



The electric field inherits the LAW OF LINEAR SUPERPOSITION from the Coulombic force:

$$\vec{F}_{q q_0} = \sum_{i=1}^{N} \vec{F}_{q_i q_0}$$

Thus the NET electric field,

$$\vec{E}_P = \lim_{q_0 \to 0} \frac{1}{q_0} \vec{F}_{q q_0} = \sum_{i=1}^{N} \lim_{q_0 \to 0} \frac{1}{q_0} \vec{F}_{q_i q_0} = \sum_{i=1}^{N} \vec{E}_{q_i} = \sum_{i=1}^{N} k \frac{q_i}{r^2_{i0}} \hat{r}_{i0}$$

In our trivial examples of electric fields examined, Coulomb's Law is just as useful a calculational tool.

Q: So, why introduce the Electric Field?

A: There are many situations (*distributions* of charge, for example) in which it is too hard to use Coulomb's Law, but we CAN determine the electric field. We also realise that the electric force exerted **on** a particle with charge q in an electric field $\vec{E}$ is

$\vec{F}_q = q\,\vec{E}$ *regardless of how the* $\vec{E}$*-field is produced.*