The Nature of Inhabited Planets and their Inhabitants

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Earth-like planets are expected to provide the greatest opportunity for the detection of life beyond the Solar System. This notion stems from an assumption that the Earth constitutes a simple random sample amongst inhabited planets. However, in the event that other intelligent species exist, our planet should not be considered a fair sample. Just as a person's country of origin is a biased sample among countries, so too their planet of origin is a biased sample among planets. The strength of this effect can be substantial: over 98% of the world's population live in a country larger than the median.

Any variable which influences either the population size or birth rate is susceptible to selection bias. In the context of a simple model where the mean population density is invariant to planet size, we infer that an inhabited planet selected at random (such as our nearest neighbour) has a radius $r < 1.2r_{\oplus}$ (95% confidence bound). If the range of habitable radii is sufficiently broad, most inhabited planets are likely to be closer in size to Mars than the Earth. Furthermore, since population density is widely observed to decline with increasing body mass, we conclude that most intelligent species are expected to exceed 300 kg.

Primitive life-forms are a pre-requisite for advanced life, and so the planets which host them must trace at least the same volume of parameter space. Our conclusions are therefore not restricted to the search for intelligent life, but may be of significance when surveying exoplanets for atmospheric biomarkers.

Introduction.— The discovery of extra-terrestrial life stands as one of the most ambitious objectives in modern scientific endeavour. Over one thousand distant planets have now been identified, spanning a broad spectrum of sizes and orbital configurations [1–4]. Since many more await detection, only a small fraction can be subject to detailed follow-up investigations. We must therefore identify those deemed most likely to host life. At present the Earth is our only example of an inhabited planet, so its physical characteristics appear to provide a natural template for finding life elsewhere. However, as we shall see, selection effects may have biased our observational sample.

It has often been postulated that our existence in the Universe could explain the magnitudes of various quantities in fundamental physics, such as the fine-structure constant, the cosmological constant and primordial density perturbations [5–13]. If an ensemble of cosmological conditions exists, we should expect to observe those which permit the emergence of life. Or more specifically, those which maximise the abundance of life. While this work will follow a similar line of reasoning, our approach differs from most in that there is no requirement for an ensemble of universes to exist. The ensemble in question is a local one, the Milky Way.

The physical characteristics of the Earth are considered to be the gold standard for habitability [14]. However, for any non-singular distribution of population sizes, typical beings do not live within typical populations. This is a statistical truism, yet it implies a violation of the mediocrity principle is inevitable. The implication that we should expect to be a member of a large civilisation has been mentioned previously by Gott [15]. Here we elaborate on this by quantifying the expected magnitude of this bias, and the bias induced in correlated variables such as the radius of the host planet.

Population Bias.— First we assess the relative likelihood of finding an individual within a given population size. Consider a game show in which there are two rooms: Room A contains N people, while Room B holds 10N. Contestants are obliged to place a bet of \$1 on which room they think they are in, and will win \$1 if correct.

An intuitive calculation states that Room B is ten times more likely than Room A, since it holds ten times more people. A counter-argument asserts that you were more likely to appear on the game show in the first place if N were larger, since there would be more contestants in total. This would precisely cancel the room allocation bias, so the relative likelihoods of Room A or Room B should be equal. But consider this scenario from the perspective of the game show's producers. If all contestants adopt the first algorithm and place their bets on Room B, they will make a collective net profit of 9N. Conversely if contestants follow the second algorithm, they are equally likely to bet on A or B, so the expected net profit is zero.

The first algorithm is clearly preferable, but why was the second algorithm unsuccessful? For any finite range of N values, the prior must be truncated at N_{max} . It is this truncation which breaks the symmetry of the argument, because if we find ourselves in a room exceeding N_{max} people, then we must be in Room B.

This population selection effect is also apparent with respect to our geographic location. It seems unremarkable to note that you are much more likely to be living in China than the Cayman Islands. But if humans had colonised other planets, the likelihood of finding yourself on a given planet must be weighted in the same manner. A second extension to this concept is of particular importance to this work: if the colony had not arrived from Earth but instead evolved independently, it appears difficult to justify why our calculation should no longer hold.

Throughout this work we shall assume that the Universe hosts an ensemble of inhabited planets. We define an advanced civilisation as one which has (a) colonised most of its host planet/moon and (b) developed sufficient intelligence to contemplate the existence of other inhabited planets. It is helpful to express the total number of observers N generated by a given civilisation in the form

$$N = \frac{xL}{R} \,. \tag{1}$$

Here L is the longevity of the civilisation, over which time it sustains a mean population size x. The mean lifespan averaged over all individuals is denoted by R. No assumption is made regarding the temporal variation of the population size or birth rate. If the ensemble of civilisations is suitably large we may consider a continuous probability distribution for the set of civilisation characteristics $\mathbf{C} \equiv \{x, L, R, \ldots\}$. The conditional probability of a civilisation \mathbf{C} sampled from an ensemble at a fixed time T differs from one sampled by an individual I, as given by Bayes' theorem

$$p(\mathbf{C}|I) \propto \frac{x}{R} p(\mathbf{C}|T)$$
. (2)

Therefore sampling civilisations via an individual disproportionally favours those with a higher population size [15] and lower mean lifespans compared to other co-existing civilisations. This is true for *all* probability density functions with non-zero variance, a brief proof is included in the Appendix.

In accordance with the central limit theorem, many complex phenomena are well described by a lognormal distribution, such the abundance of species and their physical dimensions [16, 17]. If the distribution of mean population sizes among co-existing civilisations, p(x|T) is a lognormal $\ln \mathcal{N}(\mu_x, \sigma_x^2)$, then the population-weighted p(x|I) is given by $\ln \mathcal{N}(\mu_x + \sigma_x^2, \sigma_x^2)$. The median is therefore enhanced by a factor of exp (σ_x^2) . On Earth, the abundance of species exhibits a value of $\sigma \simeq 2.4$ [16]. If the level of inter-planetary biodiversity matches or exceeds this intra-planetary distribution, the typical advanced civilisation can be expected to comprise of less than fifty million individuals.

Planetary Selection Effects.— We wish to estimate the physical characteristics of currently inhabited planets $p(\mathbf{P}|T)$, where \mathbf{P} encompasses variables such as size, density, and atmospheric properties. Following a similar

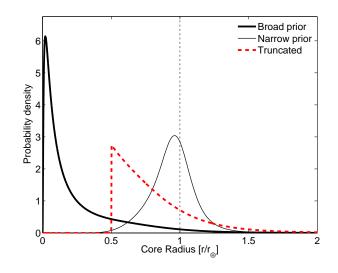


FIG. 1. Constraints on the radius r of an inhabited planet, based on a constant mean population density, and after marginalising over the μ and σ parameters of the lognormal distribution. The thick and thin solid lines correspond to the higher and lower set of σ values. The dashed line illustrates the effect of imposing the condition $r > 0.5r_{\oplus}$, as may be the requirement for an atmospheric water cycle. A common feature is the manner in which super-Earths are disfavoured, regardless of the choice of prior. Broad and narrow lines yield 95% confidence bounds of $r < 0.9r_{\oplus}$ and $r < 1.2r_{\oplus}$ respectively, while the dashed line sets $r < 1.4r_{\oplus}$. For reference, the radii of Europa and Mars are $0.25r_{\oplus}$ and $0.53r_{\oplus}$ respectively.

method used to generate (2) and marginalising over **C** yields the relation

$$p(\mathbf{P}|I) \propto p(\mathbf{P}|T) \iint \frac{x}{R} p(x, R|\mathbf{P}, T) \mathrm{d}x \,\mathrm{d}R.$$
 (3)

Selection bias will arise provided the integrand displays some dependence on **P**. In other words, provided the mean population size of advanced civilisations is correlated with *any* planetary characteristic, then the Earth is a biased sample among inhabited planets. This is the central result of this work, which we now apply to the particular case of the planet radius r.

Using our single data point of the Earth, we seek to estimate the radius of another inhabited planet, such as our nearest neighbour. Initially we adopt a minimal set of priors, applying no restriction to the conditions required for life. Any correlation between the lifespan of individuals and the size of the host planet is considered to be negligible. We model the true distribution of radii as a lognormal, $\ln \mathcal{N}(\mu_r, \sigma_r^2)$. In general, if the conditional expectation of x follows a scaling relation $E(x|r, T) \propto r^{\alpha}$, then the population-weighted distribution of radii is specified by $\ln \mathcal{N}(\mu_r + \alpha \sigma_r^2, \sigma_r^2)$. The induced bias in r is therefore given by

$$\frac{E(r|x,I)}{E(r|x,T)} = e^{\alpha \sigma_r^2}, \qquad (4)$$

For small perturbations in radius we can expect an approximately constant mean population density, therefore we adopt $\alpha = 2$. To give a specific example, $\sigma_r = 0.5$ yields a bias factor of 1.65. That is, the mean planet volume experienced by observers would be approximately five times greater than the true mean.

To determine the full posterior likelihood p(r|D,T)of inhabited planetary radii given our data D, we marginalise over the lognormal parameters with a reference prior $\pi(\mu_r, \sigma_r^2) \propto \sigma_r^{-2}$,

$$p(r|D,T) \propto \iint p(D|\mu_r,\sigma_r)p(r|\mu_r,\sigma_r)\pi(\mu_r,\sigma_r^2)\mathrm{d}\mu_r\mathrm{d}\sigma_r^2.$$
(5)

In Figure 1 we illustrate the likelihoods resulting from two different ranges in σ_r . The thin solid line corresponds to narrow distributions $0.05 < \sigma_r < 0.2$ while the thick solid line spans $0.2 < \sigma_r < 0.8$. At larger values of σ_r , the Earth becomes highly atypical. The broad and narrow distributions generate upper bounds of $r < 0.9r_{\oplus}$ and $r < 1.2r_{\oplus}$ respectively (95% confidence bound). Smaller values of σ_r will further lower the bound on r.

Thus far we have adopted a highly agnostic viewpoint, deliberately excluding factors such as the conditions necessary for life. The smallest planets are not expected to be able to retain a thick enough atmosphere to sustain liquid water on the surface. Truncating the posterior likelihood generated from the broad prior such that $p(r < 0.5r_{\oplus}) = 0$ leads to a modest amplification of the likelihood at larger radii, as shown by the dashed line. In this case we find the 95% confidence bound to be $r < 1.4r_{\oplus}$.

Note that we have only estimated the relative abundance of inhabited planets, and make no statement regarding the overall prevalence of life in the Universe. These results are insensitive to the variance among population sizes, σ_x^2 , to the variables which appear in the Drake equation [18], and to the numerous variables which influence population size, provided they remain uncorrelated with planet size.

Characteristics of Advanced Species.— Species with a lower body mass are able to sustain a higher population density. This is a trend which has been extensively observed throughout the animal kingdom [20–22]. One proposed mechanism originates from Kleiber's law, the scaling relation linking the basal metabolic rate (BMR) to the body mass m_s , BMR $\propto m_s^{3/4}$ [23]. Given the finite energy resources available in the biosphere, population density drops as the energy demand of the individual rises. Ants and termites vastly outnumber humans due to their small size. If mankind tried to match their population, our total metabolic demand would exceed the entire solar flux incident upon our planet.

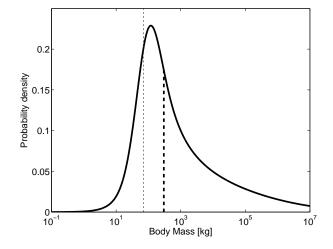


FIG. 2. One-dimensional constraint on the body mass of an intelligent species after marginalising over the two lognormal parameters. The thin dashed line represents our data point, at 70 kg, while the thick dashed line represents the median mass, at 314kg. This offset is predominantly due to the expected fall in population size with increasing body mass. For reference the adult African Elephant is approximately 6,000 kg, while the heaviest dinosaurs such as Argentinasaurus were thought to be approximately 10^6 kg [19].

We adopt a scaling relation between mean population and body mass given by $E(x|m_s,T) \propto m_s^{-3/4}$. Variations in the value of this exponent have been observed [22], however these were generally found to be steeper relationships, thus our scaling relation is a conservative one. It is also the case that larger animals live longer [24, 25], suggesting $R \propto m_s^{1/4}$. Therefore we estimate the key quantity $\langle \frac{x}{R} \rangle$ is inversely proportional to body mass.

The distribution of body masses among species on Earth can be well described by a lognormal [26]. Again we marginalise over the two parameters using a reference prior $\pi(\mu_m, \sigma_m^2) \propto \sigma_m^{-2}$. It would be extremely surprising if the inter-planetary variation in body mass is lower than that amongst a small group of closely related species on Earth. There are seven species of great ape, spanning gorillas, orangutans, humans, and chimpanzees. Their body masses exhibit an inter-species standard deviation in log space of $\sigma_m \simeq 0.5$, which serves as our lower bound. For an upper bound we adopt $\sigma_m = 3$, so as not to greatly exceed the terrestrial variance.

Figure 2 illustrates the probability density function for body mass, as derived from our single data point of 70 kg. The median body mass is found to be 314 kg, while the 95% lower bound is given by $m_s > 25$ kg. While it is likely some correlation exists between planet size and median body mass, due to the influence of surface gravity, we do not attempt to model this here.

In Figure 3 we illustrate a particular distribution with $\sigma_m = 1.22$, which is the centre of our prior in log space.

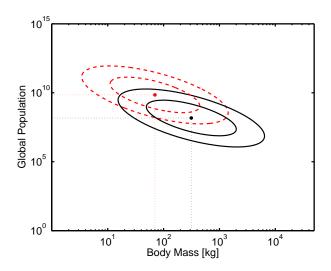


FIG. 3. An illustration of the strong bias which may be induced when sampling on a *per-individual* basis rather than a *per-population* basis. The solid contours represent where 68% and 95% of civilisations would be found in the particular case where $\sigma_m = 1.22$, while the dashed contours represent the same distribution as sampled by an individual. The median per-individual body mass is calibrated to 70 kg, representing the human species. The inner and outer contours represent where 68% and 95% of species would be found in this model.

The dashed contours show the 68% and 95% confidence limits for the mean body mass of a species as sampled by individuals. The solid lines show the same contours but when sampling by intelligent species. The offset between these two sets of contours represents the strength of the selection effect. We calibrate the axes such that the population-weighted median is 70 kg, which implies a typical advanced species of around 314 kg, matching that of the marginalised result. For this illustration we adopted $\sigma_x = \log(10)$, but this has no influence on the bias induced in m_s .

For a more detailed assessment of the variables which may influence the mean population size, x may be decomposed into physical and biological factors as quantified by

$$x = \eta_b A \frac{E_s}{E_i} \,, \tag{6}$$

where A is the available area, E_s is the energy flux density at the surface, E_i the energy demand of the individual, and η_b is the energy efficiency of the species. If the highest populations are resource limited, this may suggest we are receiving an unusually high radiation flux E_s , which can arise either from being relatively close to our host star, or by possessing a lower atmospheric opacity. In red dwarf systems most of the incident radiation is in the infrared which could lead to a considerably lower E_s .

Conclusions.— Unless we are alone in the Universe, our planet is likely to be one which produces observers at a

higher rate than most other inhabited planets. This may be accomplished by having a relatively large population, and low individual life expectancy. The magnitude of this observational bias increases exponentially with the variance of population sizes across different civilisations.

Any variable which correlates with population size will also be subject to observational bias. By adopting a simple model where the mean population density is insensitive to planet size, we find that an inhabited planet selected at random can be expected to have a radius $r < 1.2r_{\oplus}$ (95% confidence bound).

Even if we are the only intelligent species in the Universe, this does not imply that the Earth is a representative sample among planets which host life. Larger biospheres will host a wider range of species and a greater number of individual life-forms. For these reasons it seems highly likely that larger biospheres possess a greater probability of producing an intelligent species. This reinforces our conclusion that most lifebearing planets are smaller than the Earth.

The preference for smaller planets is not restricted to the search for intelligent life. Primitive life-forms are a pre-requisite for advanced life, and so their host planets must trace at least the same volume of parameter space as those of intelligent species. Two distinct methods are currently being pursued for finding life on exoplanets: biomarkers within the atmospheric spectra of exoplanets [27], and somewhat more speculatively, the reception of radio signals from advanced civilisations [28]. In each case the signal is extremely challenging to detect, and it is therefore vital to correctly prioritise the strongest prospects. Larger inhabited planets and larger populations may correspond to stronger signals, however since they are expected to be relatively scarce this gain is likely to be offset by their greater distance from the Earth.

Throughout the animal kingdom, species which are physically larger invariably possess a lower population density, possibly due to their enhanced energy demands. As a result, we should expect humans to be physically smaller than most other advanced species. By marginalising over a feasible range of standard deviations, we conclude that most species are expected to exceed 300 kg in body mass. The median body mass is similar to that of a polar bear.

While larger species possess larger brains, the correlation between brain size and intelligence is weak. Higher intelligence enables the development of technologies which can sustain larger population sizes. However it also enables the longevity of individuals R to increase, pushing the selection bias in the opposite direction. The net effect of this selection bias is therefore unclear.

The degree to which mankind and the Earth are atypical hinges on the level of diversity among advanced life forms. As we have repeatedly learned from the discoveries of distant planets, and the exploration of life on our own, nature is invariably more diverse than we anticipate, not less.

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Appendix.—

Here we present a brief proof that a typical individual expects to live within an above-median population size, for *any* choice of distribution of population sizes. We define $p(x, \mathbf{P})$ as the probability density function describing the distribution of inhabited planets in our universe with population size x and characterised by the set of physical characteristics \mathbf{P} . The probability of selecting an individual within a population size x is given by p_w

$$p_w(x, \mathbf{P}) \propto x p(x, \mathbf{P}).$$
 (7)

Provided the distribution isn't singular, all points at values of x above the median of $p(x, \mathbf{P})$ are amplified by a greater factor than any value below the median. Therefore the weighted distribution satisfies the inequality $\int_{0}^{\mu_{1/2}} p_w(x) dx < 0.5$, confirming that in general

$$Median\{p_w(x)\} > Median\{p(x)\}.$$
 (8)

Even in the case of an ensemble of universes, since (8) is true for any given universe, it also holds for all observers across all universes. Adopting either the self-sampling assumption [29] or the self-indication assumption [30], yields the same conclusion in this instance.

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- [1] K. Rice, Challenges 5, 296 (2014).
- [2] F. Pepe, C. Lovis, D. Ségransan, W. Benz, F. Bouchy, X. Dumusque, M. Mayor, D. Queloz, N. C. Santos, and S. Udry, Astron. & Astrophys. **534**, A58 (2011), arXiv:1108.3447 [astro-ph.EP].
- [3] X. Bonfils, X. Delfosse, S. Udry, T. Forveille, M. Mayor, C. Perrier, F. Bouchy, M. Gillon, C. Lovis, F. Pepe, D. Queloz, N. C. Santos, D. Ségransan, and J.-L. Bertaux, Astron. & Astrophys. 549, A109 (2013), arXiv:1111.5019 [astro-ph.EP].

- [4] E. A. Petigura, A. W. Howard, and G. W. Marcy, Proceedings of the National Academy of Science 110, 19273 (2013), arXiv:1311.6806 [astro-ph.EP].
- [5] B. Carter, in Confrontation of Cosmological Theories with Observational Data, IAU Symposium, Vol. 63, edited by M. S. Longair (1974) pp. 291–298.
- [6] B. Carter and W. H. McCrea, Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences **310**, 347 (1983).
- [7] S. Weinberg, Physical Review Letters **59**, 2607 (1987).
- [8] G. Efstathiou, Mon.Not.Roy.As.Soc. 274, L73 (1995).
- [9] A. Vilenkin, Phys. Rev. D 52, 3365 (1995).
- [10] J. A. Peacock, Mon.Not.Roy.As.Soc. 379, 1067 (2007), arXiv:0705.0898.
- [11] T. Piran and R. Jimenez, Physical Review Letters 113, 231102 (2014), arXiv:1409.2506 [astro-ph.HE].
- [12] M. Tegmark and M. J. Rees, The Astrophysical Journal 499, 526 (1998).
- [13] M. Tegmark, A. Aguirre, M. J. Rees, and F. Wilczek, Physical Review D 73, 023505 (2006).
- [14] J. F. Kasting, D. P. Whitmire, and R. T. Reynolds, Icarus **101**, 108 (1993).
- [15] J. R. Gott, Nature **363**, 315 (1993).
- [16] F. W. Preston, Ecology 29, 254 (1948).
- [17] A. E. Magurran, *Measuring biological diversity* (John Wiley & Sons, 2013).
- [18] F. Drake and D. Sobel, *Is anyone out there?* (Delacorte Press, 1992).
- [19] W. I. Sellers, L. Margetts, R. A. Coria, and P. L. Manning, PloS one 8, e78733 (2013).
- [20] J. Damuth, Nature **290**, 699 (1981).
- [21] J. Damuth, Biological Journal of the Linnean Society 31, 193 (1987).
- [22] N. Loeuille and M. Loreau, Ecology Letters 9, 171 (2006).
- [23] P. S. Agutter and D. N. Wheatley, Theoretical Biology and Medical Modelling 1, 13 (2004).
- [24] J. R. Speakman, Journal of Experimental Biology 208, 1717 (2005).
- [25] A. Hulbert, R. Pamplona, R. Buffenstein, and W. Buttemer, Physiological Reviews 87, 1175 (2007).
- [26] J. J. Greenwood, R. D. Gregory, S. Harris, P. A. Morris, and D. W. Yalden, Philosophical Transactions of the Royal Society of London. Series B: Biological Sciences 351, 265 (1996).
- [27] P. Hedelt, P. von Paris, M. Godolt, S. Gebauer, J. L. Grenfell, H. Rauer, F. Schreier, F. Selsis, and T. Trautmann, Astron. & Astrophys. 553, A9 (2013), arXiv:1302.5516 [astro-ph.EP].
- [28] J. Tarter, R. Ackermann, W. Barott, P. Backus, M. Davis, J. Dreher, G. Harp, J. Jordan, T. Kilsdonk, S. Shostak, *et al.*, Acta Astronautica **68**, 340 (2011).
- [29] N. Bostrom, The Journal of philosophy , 607 (2002).
- [30] N. Bostrom and M. M. Ćirković, The Philosophical Quarterly 53, 83 (2003).